Volatilities in High Dimension

A Two-Step General Dynamic Factor Model Approach

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This talk:

1. a two-step general dynamic factor procedure to estimate the common return and volatility factors from a large panel (a high-dimensional time series) of stock returns
2. yielding one-step-ahead conditional quantiles (VaRs) and prediction intervals for returns;
3. yielding a detailed analysis (impulse response functions etc.) of the propagation of market volatility shocks across returns;
4. the approach is non-parametric and model-free;
5. comparison with more standard parametric GARCH-type methods.

Motivation:

- Typically, parametric Value at Risk measures are built using parametric estimates of the volatility of returns
  Francq and Zakoian, 2010;

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- Typically, Values at Risk and prediction intervals are built from parametric estimates of the volatility of returns
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- but parametric multivariate volatility models in high (also moderate) dimensions run into severe curse of dimensionality problems;

- Factor models decompose a high-dimensional time series (typically, in this context, returns) into a **common component** driven by a **small number of market (common) shocks** and an **idiosyncratic component** which is only **mildly cross-correlated**;

- but being entirely (unconditional) covariance-based, a factor model for returns does not tell us anything about **volatilities**.

Motivation:

- Typically, Value at Risk measures are estimated by fitting some some parametric volatility models on observed returns
  Francq and Zakoian, 2010;
- but parametric multivariate volatility models in high (also moderate) dimensions run into severe curse of dimensionality problems;
- Factor models decompose returns into a **common**, market-driven, component and an **idiosyncratic** one, turning the curse of dimensionality into a blessing;
- but being entirely (unconditional) covariance-based, factor models do not say anything about **volatilities**;
- ... combining these approaches sounds like a good idea ...

A very natural way is the one adopted in a variety of “Factor-GARCH" methods:

- Run a Dynamic Factor Model step on the high-dimensional series of returns, disentangling the common and idiosyncratic components of returns;
- extract the low-dimensional shocks driving the common component of returns;
- perform a parametric GARCH-type analysis of those common shocks (no curse of dimensionality).

The idea at first sight looks simple and natural. It is based on the postulate

common volatility shock (market volatility shock)
$=$ shock to the volatility of the common components of returns

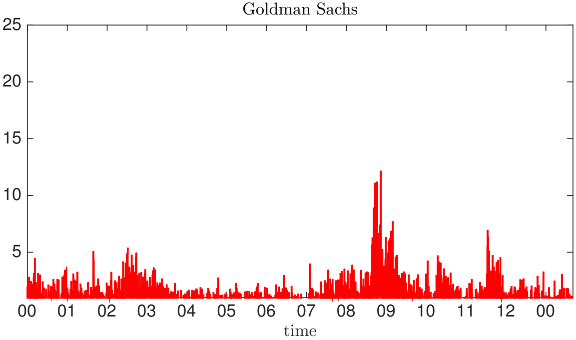
This simple and natural idea boils down to defining "market risk" as the risk associated with the market-driven component of returns (the common components):

market risk := risk of the market-driven component of returns

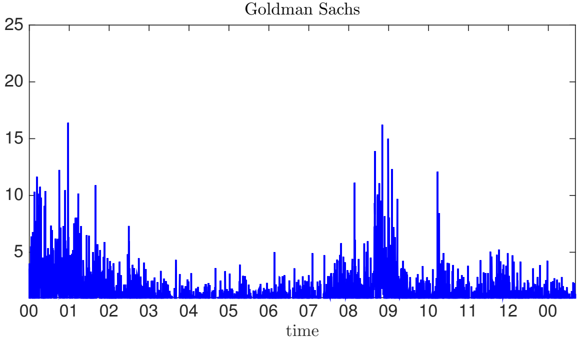
This approach is quite common—see, for instance, Fan, Liao, and Shi (2013) where the "market risk" is defined as the covariance matrix $\boldsymbol{\Sigma}_{\text{common}} = \text{Cov}(\mathbf{X})$ of the common component (in a low rank + sparse context).

How reasonable is that idea?

Volatility of common component of returns



Volatility of idiosyncratic component of returns



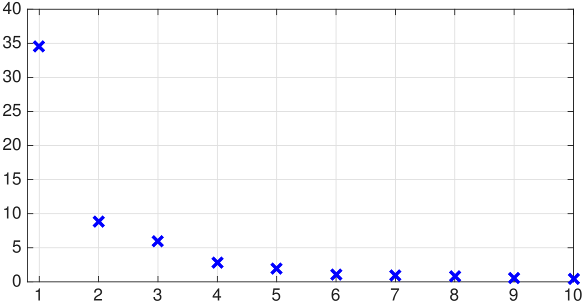
- idiosyncratic component volatility has the same magnitude as common component volatility
- idiosyncratic component volatility and common component volatility obviously strongly comove ... hence idiosyncratic component volatility is not idiosyncratic! market volatility shocks are impacting the level-idiosyncratic components as much as they do the level-common ones

Not a big surprise: the decomposition between level-common and level-idiosyncratic indeed is based on the autocovariance structure of levels only, which carries no information on volatilities.

- Actually, the empirical evidence of a factor model structure for log-volatilities is as strong as for the factor model structure of the returns

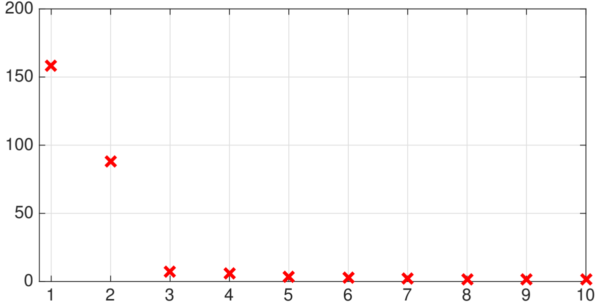
Dynamic factor structure of returns

scree-plot of eigenvalues of long-run covariance matrix of returns (S&P100)



Dynamic factor structure of log-volatilities

scree-plot of eigenvalues of long-run covariance matrix of log-volatilities (S&P100)



See Herskovic, Kelly, Lustig, Van Nieuwerburgh (2016) for further evidence.

This strongly suggests considering the two-step approach we now describe.

References:

1. Factor GARCH (among many others)

   Diebold and Nerlove (1989), Engle, Ng, Rotschild (1990), Sentana, Calzolari, Fiorentini (2008) [fitting a parametric GARCH-type model to the common shocks] Trucíos, Mazzeu, Zevallos, Hallin, Hotta, Valls Pereira (2019) [fitting a parametric GARCH-type model to the common shocks and univariate parametric AR-GARCHs to idiosyncratic components]

2. Common factor in idiosyncratic volatility

   Herskovic, Kelly, Lustig, Van Nieuwerburgh (2016) [idiosyncratic volatility exhibits a strong factor structure]

3. Two-step factors

   - Barigozzi and Hallin (2016) "Generalized Dynamic Factor Models and Volatilities: Recovering the Market Volatility Shocks", *The Econometrics Journal*
   - Barigozzi and Hallin (2017) "Generalized Dynamic Factor Models and Volatilities: Estimation and Forecasting", *Journal of Econometrics*
   - Barigozzi and Hallin (2017) "A Network Analysis of the Volatility of High-Dimensional Financial Series", *Journal of the Royal Statistical Society - series C*
   - Barigozzi, Hallin, and Soccorsi (2018) "Identification of Global and Local Shocks in International Financial Markets via General Dynamic Factor Models", *Journal of Financial Econometrics*

A two-step GDFM approach

Consider an $n \times T$ panel of stock returns (or levels)

$$\mathbf{Y}_{nt} = \{Y_{it} | i = 1, \ldots n, \; t = 1, \ldots, T\}$$

a finite realization of the stochastic process $\{Y_{it} | i \in \mathbb{N}, \; t \in \mathbb{Z}\}$.

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To capture all interdependencies in $\mathbf{Y}_n$ parametric methods are quite helpless:

curse of dimensionality!

If $n \sim 100$ we need about 10^4 parameters for linear dependencies only, plus (at least) another 10^4 parameters necessary for modelling, e.g., dependencies in the squares (volatility) ...

Factor model methods allow for dimension reduction in returns:

$$\mathbf{Y}_{nt} = \underbrace{\mathbf{X}_{nt}}_{\textit{common}} + \underbrace{\mathbf{Z}_{nt}}_{\textit{idiosyncratic}}$$

1. $\mathbf{X}_{nt}$ driven by $q \ll n$ factors, **reduced rank** spectral density;
2. $\mathbf{Z}_{nt}$ has n components which are only **weakly** *cross*-**correlated**.

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2. $\mathbf{Z}_{nt}$ has n components which are only **weakly** *cross*-**correlated**.

As $n \to \infty$, $\mathbf{X}_{nt}$ and $\mathbf{Z}_{nt}$ are identified by means of adequate (dynamic) cross-sectional averaging:

blessing of dimensionality!

Therefore, (n, T)-asymptotics are considered throughout.

- A "divide and rule" strategy:

  –Being reduced rank, the series of common components somehow can be handled as a low-dimensional series—in particular, the (low-dimensional) common shocks can be recovered and fundamental representations of the X_{it}'s can be estimated

  –Being only mildly cross-correlated, the n-dimensional series of idiosyncratic components Z_{it} can be handled, without much loss, as n univariate (auto-correlated but not cross-correlated) series. In particular, univariate AR fits and a global VAR fit roughly produce the same residuals

- The GDFM decomposition (contrary to the static factor model) is a representation result—not really a statistical *model*

Let the returns admit the GDFM representation (encompasses all other factor models considered in the literature)

$$Y_{it} - \mathrm{E}[Y_{it}] = X_{it} + Z_{it} = \sum_{j=1}^{q} \sum_{k=0}^{\infty} b_{ijk} u_{jt-k} + \sum_{k=0}^{\infty} d_{ik} v_{it-k}$$

or

$$\mathbf{Y}_{nt} - \mathrm{E}[\mathbf{Y}_{nt}] = \mathbf{X}_{nt} + \mathbf{Z}_{nt} = \underbrace{\mathbf{B}_n(L)\mathbf{u}_t}_{common} + \underbrace{\mathbf{D}_n(L)\mathbf{v}_{nt}}_{idiosyncratic}$$

such that

- **L1** $\mathbf{u}_t$ is 2nd-order q-dim white noise, zero mean, with diagonal covariance;
- **L2** $\mathbf{B}_n(L)$ is rational and has absolutely summable coefficients;
- **L3** the q spectral eigenvalues of $\mathbf{X}_{nt}$ diverge linearly in n (a reduced-rank spectrum);

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such that

- **L4** $\mathbf{v}_{nt}$ is 2nd-order n-dim white noise, zero mean, with p.d. covariance, and such that its largest eigenvalue is bounded uniformly in n;
- **L5** $\mathrm{E}[v_{it}|v_{is}] = 0$ for all i and $t > s$;
- **L6** $\mathbf{D}_n(L)$ diagonal, and has absolutely summable coefficients; $d_i(L) = c_i^{-1}(L)$ with $c_i(L)$ of finite order and $c_i(z) \neq 0$ for $|z| \leq 1$, i.e. $c_i(L)Z_{it} = v_{it}$;
- **L7** the largest spectral eigenvalue of $\mathbf{Z}_{nt}$ is bounded uniformly in n

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$$\mathbf{Y}_{nt} - \mathrm{E}[\mathbf{Y}_{nt}] = \mathbf{X}_{nt} + \mathbf{Z}_{nt} = \underbrace{\mathbf{B}_n(L)\mathbf{u}_t}_{common} + \underbrace{\mathbf{D}_n(L)\mathbf{v}_{nt}}_{idiosyncratic}$$

such that

L8 $\mathrm{Cov}(u_{jt}, v_{is}) = 0$ for any i, j, t, s;

L9 $\mathbf{u}_t$ and $\mathbf{v}_{nt}$ have finite fourth-order cumulants.

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$$\mathbf{Y}_{nt} - \mathrm{E}[\mathbf{Y}_{nt}] = \mathbf{X}_{nt} + \mathbf{Z}_{nt} = \underbrace{\mathbf{B}_n(L)\mathbf{u}_t}_{common} + \underbrace{\mathbf{D}_n(L)\mathbf{v}_{nt}}_{idiosyncratic}$$

such that
in terms of the observed Y_{it}'s,

- the q largest spectral eigenvalues of $\mathbf{Y}_{nt}$ diverge linearly in n;
- the $(q+1)$-th largest spectral eigenvalue of $\mathbf{Y}_{nt}$ is bounded uniformly in n.

Alternative representation (Forni, Hallin, Lippi, and Zaffaroni, 2015, 2017)

$$(\mathbf{I}_n - \mathbf{A}_n(L))\,(\mathbf{Y}_{nt} - \mathrm{E}[\mathbf{Y}_{nt}]) = \mathbf{H}_n\mathbf{u}_t + (\mathbf{I}_n - \mathbf{A}_n(L))\,\mathbf{Z}_{nt}$$

- $\mathbf{A}_n(L)$ is a block-diagonal matrix of one-sided finite-order filters, with blocks of size $(q+1)$;
- $\mathbf{Z}^*_{nt} := (\mathbf{I}_n - \mathbf{A}_n(L))\,\mathbf{Z}_{nt}$ is idiosyncratic;
- $\mathbf{H}_n$ is $n \times q$ with rank q;
- $\mathbf{e}_{nt} := \mathbf{H}_n\mathbf{u}_t$ is 2nd-order n-dim white noise, zero mean, with rank q covariance.

We assume

L10 $\mathbf{H}'_n\mathbf{H}_n/n \to \mathbf{I}_q$, as $n \to \infty$.

Then, the q largest eigenvalues of the covariance of $\mathbf{e}_{nt}$ diverge linearly in n.

A *static* factor model for $(\mathbf{I}_n - \mathbf{A}_n(L))\,(\mathbf{Y}_{nt} - \mathrm{E}[\mathbf{Y}_{nt}])$.

To compute volatilities we need:

(i) the innovations of Y_{it}

(ii) a non-linear transformation thereof.

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Innovations:

(i.a) the components e_{it} of $\mathbf{e}_{nt}$ (recall $\mathbf{e}_{nt} := \mathbf{H}_n \mathbf{u}_t$) are the innovations of the common component of returns;

(i.b) the components v_{it} of $\mathbf{v}_{nt}$ are the innovations of the idiosyncratic component of returns;

(i.c) since common and idiosyncratic components are mutually orthogonal (all leads and lags), let $s_{it} := e_{it} + v_{it}$.

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(i.c) since common and idiosyncratic components are mutually orthogonal (all leads and lags), let $s_{it} := e_{it} + v_{it}$.

As a proxy for log-volatilities, define (Engle and Marcucci, 2006)

$$h_{it} := \log\{(s_{it})^2\};$$

we assume that

V0 $|s_{it}| > 0$ almost surely for all i, t.

The log-volatilities h_{it} admit a GDFM representation:

$$h_{it} - \mathrm{E}[h_{it}] = \chi_{it} + \xi_{it} = \sum_{j=1}^{Q} \sum_{k=0}^{\infty} f_{ijk} \varepsilon_{jt-k} + \sum_{k=0}^{\infty} g_{ik} \nu_{it-k}$$

or

$$\mathbf{h}_{nt} - \mathrm{E}[\mathbf{h}_{nt}] = \underbrace{\boldsymbol{\chi}_{nt}}_{common} + \underbrace{\boldsymbol{\xi}_{nt}}_{idiosyncratic} = \underbrace{\mathbf{F}_n(L)\boldsymbol{\varepsilon}_t}_{common} + \underbrace{\mathbf{G}_n(L)\boldsymbol{\nu}_{nt}}_{idiosyncratic}$$

such that

V1 $\boldsymbol{\varepsilon}_t$ is 2nd-order Q-dim white noise, zero mean, with diagonal covariance;

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such that

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- **V5** $\mathrm{E}[\nu_{it}|\nu_{is}] = 0$ for all i and $t > s$;
- **V6** $\mathbf{G}_n(L)$ diagonal, and has absolutely summable coefficients;
- **V7** $g_i(L) = p_i^{-1}(L)$ with $p_i(L)$ of finite order and $p_i(z) \neq 0$ for $|z| \leq 1$, i.e. $p_i(L)\xi_{it} = \nu_{it}$;

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$$\mathbf{h}_{nt} - \mathsf{E}[\mathbf{h}_{nt}] = \underbrace{\boldsymbol{\chi}_{nt}}_{common} + \underbrace{\boldsymbol{\xi}_{nt}}_{idiosyncratic} = \underbrace{\mathbf{F}_n(L)\boldsymbol{\varepsilon}_t}_{common} + \underbrace{\mathbf{G}_n(L)\boldsymbol{\nu}_{nt}}_{idiosyncratic}$$

such that

- the largest spectral eigenvalue of $\boldsymbol{\xi}_{nt}$ is bounded for any n;
- the Q largest spectral eigenvalues of $\mathbf{h}_{nt}$ diverge linearly in n;
- the $(Q+1)$-th largest spectral eigenvalue of $\mathbf{h}_{nt}$ is bounded uniformly on n.

Alternative representation (Forni, Hallin, Lippi, and Zaffaroni, 2015, 2017)

$$(\mathbf{I}_n - \mathbf{M}_n(L))\,(\mathbf{h}_{nt} - \mathrm{E}[\mathbf{h}_{nt}]) = \mathbf{R}_n \boldsymbol{\varepsilon}_t + (\mathbf{I}_n - \mathbf{M}_n(L))\,\boldsymbol{\xi}_{nt}$$

- $\mathbf{M}_n(L)$ a block diagonal matrix of one-sided finite-order filters, with blocks of size $(Q+1)$;
- $\boldsymbol{\xi}^*_{nt} := (\mathbf{I}_n - \mathbf{M}_n(L))\,\boldsymbol{\xi}_{nt}$ is idiosyncratic;
- $\mathbf{R}_n$ is $n \times Q$ with rank Q.

We assume (an identification constraint)

V10 $\mathbf{R}_n'\mathbf{R}_n/n \to \mathbf{I}_Q$ as $n \to \infty$.

Then, the Q largest eigenvalues of the covariance of $\mathbf{R}_n \boldsymbol{\varepsilon}_{nt}$ diverge linearly in n.

A *static* factor model for $(\mathbf{I}_n - \mathbf{M}_n(L))\,(\mathbf{h}_{nt} - \mathrm{E}[\mathbf{h}_{nt}])$.

Summary of the model for levels

$$Y_{it} = \mathrm{E}[Y_{it}] + e_{it} + \underbrace{\sum_{k=1}^{\infty} \mathbf{b}'_{ik}\mathbf{u}_{t-k}}_{X_{it|t-1}} + v_{it} + \underbrace{\sum_{k=1}^{\infty} d_{ik} v_{it-k}}_{Z_{it|t-1}} =: Y_{it|t-1} + s_{it}$$

Summary of the model for log-volatilities $h_{it} = \log s_{it}^2$

$$h_{it} = \mathrm{E}[h_{it}] + \mathbf{f}'_{i0}\varepsilon_t + \underbrace{\sum_{k=1}^{\infty} \mathbf{f}'_{ik}\varepsilon_{t-k}}_{\chi_{it|t-1}} + \nu_{it} + \underbrace{\sum_{k=1}^{\infty} g_{ik}\nu_{it-k}}_{\xi_{it|t-1}} =: h_{it|t-1} + \omega_{it}$$

Combining the two,

$$s_{it} = \underbrace{\exp(h_{it|t-1}/2)}_{=: s_{it|t-1}} \underbrace{\exp(\omega_{it}/2)\mathrm{sign}(s_{it})}_{=: w_{it}}$$

hence

$$Y_{it} = Y_{it|t-1} + s_{it} = Y_{it|t-1} + s_{it|t-1} w_{it}$$

The lower and upper prediction bounds with confidence level $(1-\alpha) \in (0,1)$ are

$$\mathcal{L}_{it|t-1}(\alpha) = Y_{it|t-1} + s_{it|t-1}\ q(\alpha; w_i)$$
$$\mathcal{U}_{it|t-1}(\alpha) = Y_{it|t-1} + s_{it|t-1}\ q(1-\alpha; w_i)$$

where $q(\alpha; w_i)$ stands for the α-quantile of w_{it}.
The equal-tails prediction interval with coverage probability $(1-\alpha)$ is

$$\mathcal{I}_{it|t-1}(\alpha) = [\mathcal{L}_{it|t-1}(\alpha/2), \mathcal{U}_{it|t-1}(\alpha/2)]$$

Unequal-tails prediction intervals are also possible.

Estimation

The decomposition

$$(\mathbf{I}_n - \mathbf{A}_n(L))\,(\mathbf{Y}_{nt} - \mathrm{E}[\mathbf{Y}_{nt}]) =: \mathbf{e}_{nt} + \mathbf{Z}^*_{nt}$$

is a static factor model representation for $\mathbf{Y}^*_{nt} := (\mathbf{I}_n - \mathbf{A}_n(L))\,(\mathbf{Y}_{nt} - \mathrm{E}[\mathbf{Y}_{nt}])$

Estimation in a nutshell

- Spectral density matrix of $\mathbf{X}_{nt}$ by dynamic PCA from spectral density of $\mathbf{Y}_{nt}$;
- Autocovariances of $\mathbf{X}_{nt}$ by inverse Fourier transform;
- Yule-Walker equations on VARs of dimension $(q+1)$ to get $\widehat{\mathbf{A}}_n(L)$;
- static PCA on $(\mathbf{I}_n - \widehat{\mathbf{A}}_n(L))\mathbf{Y}_{nt}$ to get $\widehat{\mathbf{e}}_{nt}$ and $\widehat{\mathbf{Z}}_{nt}$;
- univariate AR on $\widehat{Z}_{it}$ to get $\widehat{c}_i(L)$ and $\widehat{v}_{it}$;
- estimate the GDFM of $\widehat{h}_{it} = \log(\widehat{e}_{it} + \widehat{v}_{it})^2$ as before;
- q and Q via information criteria on spectral eigenvalues (Hallin and Liška, 2007)

We add the assumptions:

- **KL** $B_T = o(\sqrt{T})$, i.e. the bandwidth for estimating the spectrum of $\mathbf{Y}_n$;
- **KV** $M_T = o(\sqrt{T})$, i.e. the bandwidth for estimating the spectrum of $\mathbf{h}_n$;
- **TL** $\mathbf{u}_t$ is sub-exponential and $\boldsymbol{w}_n'\mathbf{Z}_{nt}$ is sub-exponential for $\|\boldsymbol{w}_n\| = 1$;
- **TV** $\boldsymbol{\varepsilon}_t$ is sub-exponential and $\boldsymbol{w}_n'\boldsymbol{\xi}_{nt}$ is sub-exponential for $\|\boldsymbol{w}_n\| = 1$.
- **ERG** $\{w_{it}\}$ is ergodic

By TL and TV all common and idiosyncratic components are sub-exponential

$$\mathrm{P}(|u_{jt}| > \epsilon) \leq K_1 \exp(-\epsilon K_2), \qquad j = 1, \dots, q.$$

Results can be generalized to sub-Weibull

$$\mathrm{P}(|u_{jt}| > \epsilon) \leq K_1 \exp(-\epsilon^{\vartheta} K_2), \;\; \vartheta > 0, \qquad j = 1, \dots, q.$$

For $\vartheta < 1$ we can account for extreme events.

The estimated model is

$$Y_{it} - \bar{Y}_i = \sum_{k=0}^{\bar{k}_1} \widehat{\mathbf{b}}'_{ik} \widehat{\mathbf{u}}_{t-k} + \sum_{k=0}^{\bar{k}_2} \widehat{d}_{ik} \widehat{v}_{it}$$

Proposition 1. Let $\rho_{nT} = \max(B_T/\sqrt{T}, 1/B_T, 1/\sqrt{n})$, then (under Assumptions: see the paper), under $n = O(T^\zeta)$ for some $0 < \zeta < \infty$ as $n, T \to \infty$, there exists a $q \times q$ diagonal matrix $\mathbf{J}$ with entries ± 1, such that

(a) $\max_i \|\widehat{\mathbf{b}}_{ik} - \mathbf{J}\mathbf{b}_{ik}\| = O_P(\rho_{nT})$, for all $k \le \bar{k}_1$;

(b) $\max_t \|\widehat{\mathbf{u}}_t - \mathbf{J}\mathbf{u}_t\| = O_P(\rho_{nT} \log T)$;

(c) $\max_i |\widehat{d}_{ik} - d_{ik}| = O_P(\rho_{nT} \log T)$, for all $k \le \bar{k}_2$;

(d) $\max_i \max_t |\widehat{v}_{it} - v_{it}| = O_P(\rho_{nT} \log T)$.

Let $\widehat{s}_{it} := \widehat{e}_{it} + \widehat{v}_{it}$. For some $\kappa_T > 0$, define the capped estimated log-volatility as

$$\widehat{h}_{it} := \log(\widehat{s}_{it}^2)\mathbb{I}(|\widehat{s}_{it}| \geq \kappa_T) + \log \kappa_T^2 \mathbb{I}(|\widehat{s}_{it}| < \kappa_T).$$

Assume that this capping is such that

R $\kappa_T > 0$ and the set

$$\widehat{\mathcal{T}}_T := \{t \in \{1, \ldots, T\} \mid |\widehat{s}_{it}| < \kappa_T, \text{for all } i \in \{1, \ldots, n\}\}$$

has cardinality $|\widehat{\mathcal{T}}_T| = o_P(\sqrt{T})$ as $T \to \infty$.

Capping is bounding s_{it}^2 away from zero, robustifying their log-transforms; $\kappa_T = 0$ works in practice, though.

Simulation-based results show that we can choose $\kappa_T \asymp \log^{-\alpha} T$, with $\alpha > 0$.

The estimated model is

$$\widehat{h}_{it} - \bar{\widehat{h}}_i = \sum_{k=0}^{\bar{k}_1^*} \widehat{\mathbf{f}}'_{ik} \widehat{\varepsilon}_{t-k} + \sum_{k=0}^{\bar{k}_2^*} \widehat{g}_{ik} \widehat{\nu}_{it}$$

Proposition 2. Let $\tau_{nT} = \max(B_T M_T/\sqrt{T}, M_T/\sqrt{n})$, then, under Assumptions in the paper, and if $n = O(T^\zeta)$ for some finite $\zeta > 0$ as $n, T \to \infty$, there exists a $Q \times Q$ diagonal matrix $\mathbf{S}$ with entries ± 1, s.t.

(a) $\max_i \|\widehat{\mathbf{f}}_{ik} - \mathbf{S}\mathbf{f}_{ik}\| = O_P(\tau_{nT} \log^{1+\alpha} T)$, for all $k \leq \bar{k}_1^*$;

(b) $\max_t \|\widehat{\varepsilon}_t - \mathbf{S}\varepsilon_t\| = O_P(\tau_{nT} \log^{2+\alpha} T)$;

(c) $\max_i |\widehat{g}_{ik} - g_{ik}| = O_P(\tau_{nT} \log^{2+\alpha} T)$, for all $k \leq \bar{k}_2^*$;

(d) $\max_i \max_t |\widehat{\nu}_{it} - \nu_{it}| = O_P(\tau_{nT} \log^{2+\alpha} T)$.

Prediction

Once we have estimated the model, we can compute

1. the one-step-ahead predictions $\widehat{Y}_{iT+1|T}$ and $\widehat{s}_{iT+1|T} = \exp(\widehat{h}_{iT+1|T})$;
2. the historical innovations $\widehat{w}_{it}$ for $t = 1, \ldots, T$.

Denote by $\widehat{w}_{i(1)}, ..., \widehat{w}_{i(T)}$ the order statistic of $\widehat{w}_{i1}, ..., \widehat{w}_{iT}$.
Then $\widehat{w}_{i(\lceil T\alpha \rceil)}$ is the empirical counterpart of $q(\alpha; w_i)$.

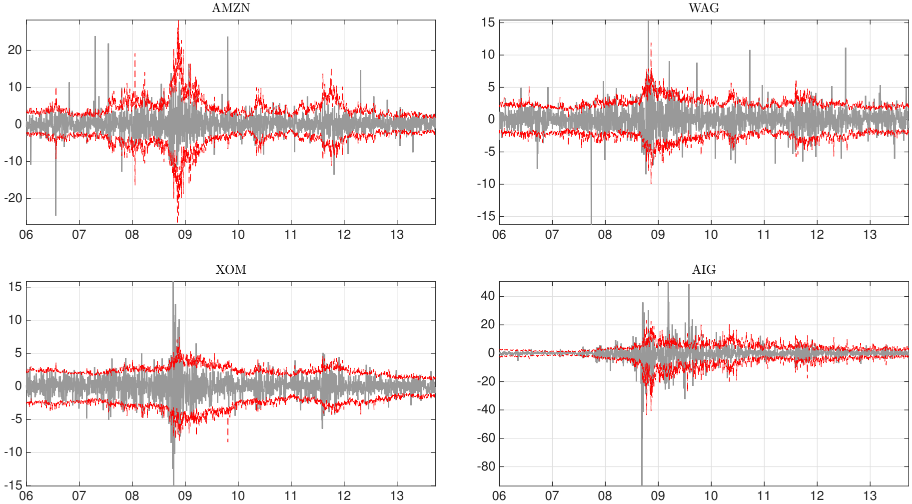
Empirical versions of prediction limits and intervals are

$$\widehat{\mathcal{L}}_{iT+1|T}(\alpha) = \widehat{Y}_{iT+1|T} + \widehat{s}_{iT+1|T} \ \widehat{w}_{i(\lceil T\alpha \rceil)}$$
$$\widehat{\mathcal{U}}_{iT+1|T}(\alpha) = \widehat{Y}_{iT+1|T} + \widehat{s}_{iT+1|T} \ \widehat{w}_{i(\lceil T(1-\alpha) \rceil)}$$
$$\widehat{\mathcal{I}}_{iT+1|T}(\alpha) = \left[\widehat{\mathcal{L}}_{iT+1|T}(\alpha/2), \widehat{\mathcal{U}}_{iT+1|T}(\alpha/2)\right]$$

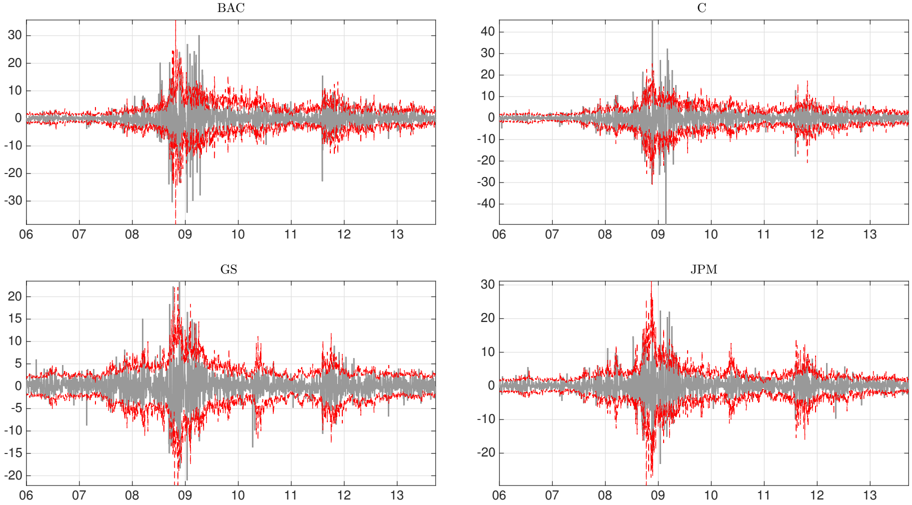
Consistent as soon as the h_{it}'s are ergodic

Data

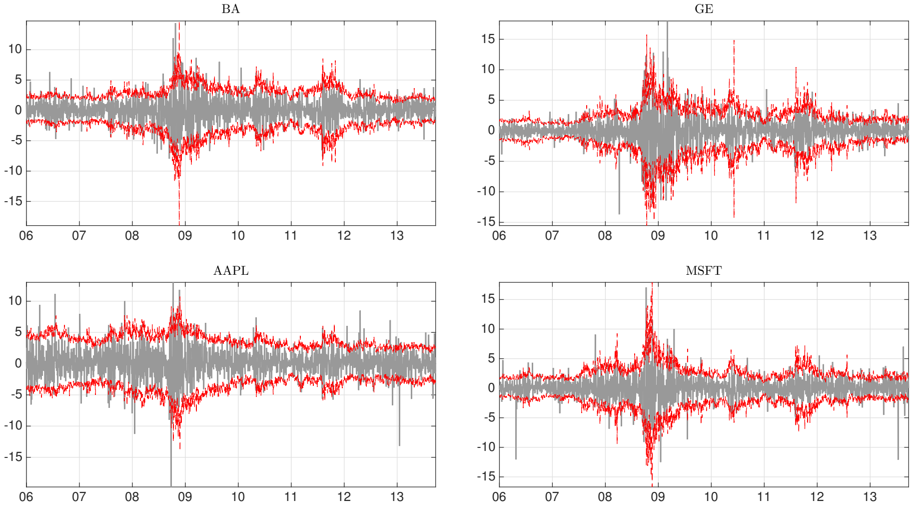
- $n = 90$ daily returns of stocks;
- from 4/1/2000 to 30/9/2013, $T = 3456$ observations;
- pseudo-out-of-sample forecasting:
  - estimating the model using $t = 1, \dots, \tau$;
  - $\tau = (T - M), \dots, (T - 1)$ and $M = 1948$;
  - evaluation period 3/1/2006 to 27/9/2013.
- number of factors for levels $q = 3$ and for log-volatilities $Q = 2$;
- bandwidth $B_T = 2$ and $M_T = 17$;
- capping constant $\kappa_T \in \{0, 0.1, 0.25, 0.5\}$;
- compute quantiles using $(\widehat{w}_{i\tau-\ell+1}, ..., \widehat{w}_{i\tau})$, with $\ell \in \{126, 252, 504, \tau\}$;
- $\alpha \in \{0.32, 0.2, 0.1, 0.05, 0.01\}$.



AMZN: Amazon; WAG: Walgreens; XOM: Exxon Mobil;
AIG: America International Group



BAC: Bank of America; C: City Group; GS: Goldman Sachs; JPM: JP Morgan Chase



BA: Boeing; GE: General Electric; AAPL: Apple; MSFT: Microsoft

Table: Average estimated coverage - GDFM

	$\kappa_T = 0$				
	$(1-\alpha)$				
	0.68	0.8	0.9	0.95	0.99
$\ell = 126$	0.6709	0.7894	0.8887	0.9400	0.9812
$\ell = 252$	0.6708	0.7903	0.8902	0.9415	0.9848
$\ell = 504$	0.6711	0.7895	0.8895	0.9412	0.9846
$\ell = \tau$	0.7010	0.8142	0.9049	0.9506	0.9881
	$\kappa_T = 0.1$				
	$(1-\alpha)$				
	0.68	0.8	0.9	0.95	0.99
$\ell = 126$	0.6874	0.7985	0.8931	0.9416	0.9813
$\ell = 252$	0.6882	0.7999	0.8940	0.9424	0.9846
$\ell = 504$	0.6886	0.7995	0.8929	0.9419	0.9843
$\ell = \tau$	0.7187	0.8244	0.9096	0.9523	0.9881
	$\kappa_T = 0.25$				
	$(1-\alpha)$				
	0.68	0.8	0.9	0.95	0.99
$\ell = 126$	0.7126	0.8141	0.8997	0.9452	0.9821
$\ell = 252$	0.7138	0.8143	0.9009	0.9452	0.9851
$\ell = 504$	0.7149	0.8150	0.9002	0.9449	0.9846
$\ell = \tau$	0.7430	0.8387	0.9162	0.9551	0.9886

Table: Average estimated coverage - univariate GARCH(1,1)s

	$(1-\alpha)$				
	0.68	0.8	0.9	0.95	0.99
$\ell = 126$	0.6755	0.7947	0.8933	0.9429	0.9834
$\ell = 252$	0.6786	0.7981	0.8968	0.9460	0.9871
$\ell = 504$	0.6807	0.7994	0.8983	0.9479	0.9878
$\ell = \tau$	0.6920	0.8077	0.9036	0.9510	0.9897

Testing for identical coverage probability of GDFM and GARCH (McNemar, 1947).

Table: Proportions of rejections in favour of a better GDFM coverage (left-hand panel), in favour of a better GARCH coverage (right-hand panel). Significance level δ. Capping $\kappa_T = 0.25$.

	better GDFM coverage			better GARCH coverage		
$\alpha = 0.1$	$\delta = 0.1$	$\delta = 0.05$	$\delta = 0.01$	$\delta = 0.1$	$\delta = 0.05$	$\delta = 0.01$
$\ell = 126$	0.60	0.54	0.37	0.10	0.08	0.07
$\ell = 252$	0.53	0.46	0.29	0.13	0.12	0.09
$\alpha = 0.05$	$\delta = 0.1$	$\delta = 0.05$	$\delta = 0.01$	$\delta = 0.1$	$\delta = 0.05$	$\delta = 0.01$
$\ell = 126$	0.41	0.28	0.16	0.11	0.08	0.06
$\ell = 252$	0.26	0.20	0.06	0.18	0.12	0.11

Backtesting (Christoffersen, 1998)

1. Define the hit-sequence: $\widehat{\mathcal{H}}_{i\tau+1|\tau}(\alpha) = \mathbb{I}(Y_{i\tau+1} \in \widehat{\mathcal{I}}_{i\tau+1|\tau}(\alpha))$.
2. Test of valid nominal coverage (reject means interval not wide enough)
$$H_{0i} : \mathsf{E}[\widehat{\mathcal{H}}^{(\ell)}_{i,\tau+1|\tau}(\alpha)] \geq (1-\alpha) \quad \text{versus} \quad H_{1i} : \mathsf{E}[\widehat{\mathcal{H}}^{(\ell)}_{i,\tau+1|\tau}(\alpha)] < (1-\alpha).$$
3. Test of sharp nominal coverage (reject means interval too wide)
$$H_{0i} : \mathsf{E}[\widehat{\mathcal{H}}^{(\ell)}_{i,\tau+1|\tau}(\alpha)] \leq (1-\alpha) \quad \text{versus} \quad H_{1i} : \mathsf{E}[\widehat{\mathcal{H}}^{(\ell)}_{i,\tau+1|\tau}(\alpha)] > (1-\alpha).$$
4. Unconditional Coverage test combining the previous ones.
5. Serial Independence test against binary first-order Markov dependence.
6. Conditional Coverage test, combining Unconditional Coverage and Independence tests.

Table: Proportion of rejections when testing for valid nominal coverage (left panel) and for sharp nominal coverage (right panel). Significance level δ. Capping $\kappa_T = 0.25$.

	valid nominal coverage test			sharp nominal coverage test		
$\alpha = 0.1$	$\delta = 0.1$	$\delta = 0.05$	$\delta = 0.01$	$\delta = 0.1$	$\delta = 0.05$	$\delta = 0.01$
$\ell = 126$	0.14	0.12	0.09	0.24	0.14	0.03
$\ell = 252$	0.16	0.13	0.08	0.31	0.21	0.09
$\alpha = 0.05$	$\delta = 0.1$	$\delta = 0.05$	$\delta = 0.01$	$\delta = 0.1$	$\delta = 0.05$	$\delta = 0.01$
$\ell = 126$	0.29	0.19	0.13	0.01	0.00	0.00
$\ell = 252$	0.30	0.20	0.14	0.06	0.01	0.00

Table: Proportion of rejections when considering the two-sided test. Significance level δ. Capping $\kappa_T = 0.25$.

	unconditional coverage test		
$\alpha = 0.1$	$\delta = 0.1$	$\delta = 0.05$	$\delta = 0.01$
$\ell = 126$	0.27	0.19	0.08
$\ell = 252$	0.34	0.26	0.13
$\alpha = 0.05$	$\delta = 0.1$	$\delta = 0.05$	$\delta = 0.01$
$\ell = 126$	0.19	0.16	0.10
$\ell = 252$	0.21	0.16	0.13

Table: Proportion of rejections when testing against serial dependence (left panel) and in the combined problem (right panel). Significance level δ. Capping $\kappa_T = 0.25$.

	independence test			conditional coverage test		
$\alpha = 0.1$	$\delta = 0.1$	$\delta = 0.05$	$\delta = 0.01$	$\delta = 0.1$	$\delta = 0.05$	$\delta = 0.01$
$\ell = 126$	0.32	0.22	0.08	0.37	0.22	0.12
$\ell = 252$	0.40	0.36	0.19	0.49	0.42	0.26

	independence test			conditional coverage test		
$\alpha = 0.05$	$\delta = 0.1$	$\delta = 0.05$	$\delta = 0.01$	$\delta = 0.1$	$\delta = 0.05$	$\delta = 0.01$
$\ell = 126$	0.28	0.20	0.06	0.28	0.21	0.12
$\ell = 252$	0.37	0.27	0.17	0.36	0.27	0.22

Summary

Based on GDFM techniques, we are able to construct nonparametric and model-free quantile-related one-step ahead prediction intervals for returns incorporating dynamic information about volatilities while escaping the curse of dimensionality.

But there's more in the two-step approach than interval predition!

Analyzing the Market Volatility Shocks

At the end of Step 1 of our two-step factor model approach, we had disentangled common and idiosyncratic shocks e_{it} and v_{it}, estimated by $widehate_{it}$ and $\widehat{v}_{it}$, respectively.

For simplicity, let us drop hats whenever we can.

- The e_{it}'s are the residuals we need for an analysis of the volatility of the level-common components. They are a reduced-rank process (dimension n, driven by q-dimensional noise).

- The v_{it}'s are the residuals we need for an analysis of the volatility of the level-idiosyncratic components.

Instead of aggregating them into $s_{it} := e_{it} + v_{it}$ ($\widehat{s}_{it} := \widehat{e}_{it} + \widehat{v}_{it}$)—which was fine for prediction purpose, let us keep both of them, and define, as proposed by Engle and Marcucci (2006),

$$h_{it}^{\mathbf{com}} := \log e_{it}^2 \qquad h_{it}^{\mathbf{idio}} := \log v_{it}^2.$$

$$h_{it}^{\text{com}} := \log e_{it}^2 \quad \text{and} \quad h_{it}^{\text{idio}} := \log v_{it}^2$$

... two panels of volatility proxies, thus, impacted by, and hence containing information on, the same market volatility shocks we are inerested in.

Those two (large) panels of residuals have to be analyzed jointly, as one $2n \times T$ panel with two $n \times T$ subpanels.

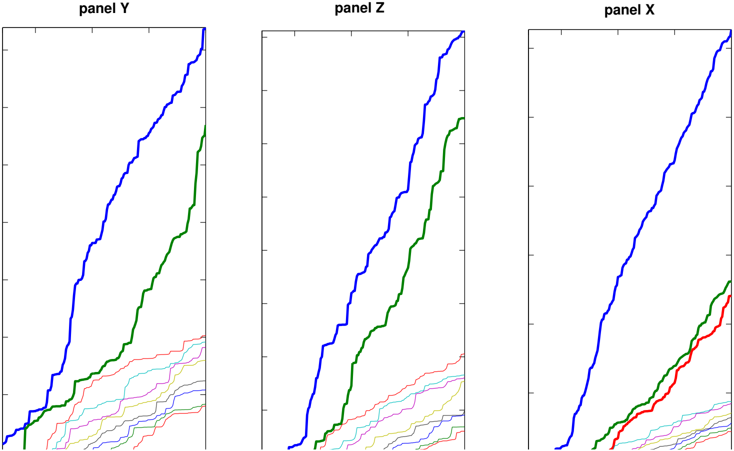
Panels with block structure have been described and studied in Hallin and Liška (*Journal of Econometrics* 2011).

For ease of presentation, consider the following example of a panel composed of two blocks: n_F=96 French economic series $\{X_{it}^F\}$ and n_G=114 German ones $\{X_{jt}^G\}$; the joint panel thus has (n= 210) series.

Subpanel spectral eigenvalues (two subpanels or blocks)

Behavior of 10 largest dynamic eigenvalues (averaged over frequencies):

(a) France; (b) Germany; (c) France and Germany.



Three distinct analyses can be conducted, based on

- two *marginal factor models*, with q_F and q_G common shocks, respectively

$$\begin{aligned} X_{it}^F &= \chi_{it}^F + \xi_{it}^F \\ X_{jt}^G &= \chi_{jt}^G + \xi_{jt}^G \end{aligned}$$

- a *global factor model*, with q common shocks

$$\begin{aligned} X_{it}^F &= \chi_{it}^{FG} + \xi_{it}^{FG} \\ X_{jt}^G &= \chi_{jt}^{FG} + \xi_{jt}^{FG} \end{aligned}$$

This provides three decompositions of the Hilbert space $\mathcal{H}$ spanned by the panel into

- an F-common space $\mathcal{H}_F^\chi$ and an F-idiosyncratic space $\mathcal{H}_F^\xi := (\mathcal{H}_F^\chi)^\perp$
- a G-common space $\mathcal{H}_G^\chi$ and a G-idiosyncratic space $\mathcal{H}_G^\xi := (\mathcal{H}_G^\chi)^\perp$
- an FG-common space $\mathcal{H}_{FG}^\chi$ and an FG-idiosyncratic space $\mathcal{H}_{FG}^\xi := (\mathcal{H}_{FG}^\chi)^\perp$

Clearly, $\mathcal{H}^{\chi}_F \subseteq \mathcal{H}^{\chi}_{FG}$ and $\mathcal{H}^{\chi}_G \subseteq \mathcal{H}^{\chi}_{FG}$ so that $\max(q_F, q_G) \leq q \leq q_F + q_G$.

We thus have two decompositions into four mutually orthogonal components:

$$X^F_{it} = \overbrace{\underbrace{\phi_{F;it} + \psi_{F;it}}_{\chi^F_{it}} + \zeta_{F;it}}^{\chi^{FG}_{it}} \hspace{-2.5em} \underbrace{\phantom{\zeta_{F;it}} + \xi^{FG}_{it}}_{\xi^F_{it}}, \quad i \in \mathbb{N},\ t \in \mathbb{Z}$$

and

$$X^G_{jt} = \overbrace{\underbrace{\phi_{G;jt} + \psi_{G;jt}}_{\chi^G_{jt}} + \zeta_{G;jt}}^{\chi^{GF}_{jt}} \hspace{-2.5em} \underbrace{\phantom{\zeta_{G;jt}} + \xi^{GF}_{jt}}_{\xi^G_{jt}}, \quad j \in \mathbb{N},\ t \in \mathbb{Z}.$$

$\phi_{F;it}$ is F- and G-common: strongly common

$\psi_{F;it}$ is F-common but G-idiosyncratic: weakly F-common

$\zeta_{F;it}$ is F-idiosyncratic but G-common: weakly F-idiosyncratic

ξ_{it}^{FG} is FG-idiosyncratic: strongly idiosyncratic

$\phi_{G;it}$ is F- and G-common: strongly common

$\psi_{G;it}$ is G-common but F-idiosyncratic: weakly G-common

$\zeta_{G;it}$ is G-idiosyncratic but F-common: weakly G-idiosyncratic

ξ_{it}^{GF} is FG-idiosyncratic: strongly idiosyncratic

Statistical analysis:

identification of q_F, q_G and q via the Hallin-Liška (*JASA* 2007) method

consistent reconstruction of $\phi_{F;it}$, $\psi_{F;it}$, etc. and estimation of their contributions to the total sum of squares as in Hallin and Liška (*Journal of Econometrics* 2011)

In our case,

the role of France is played by the $\{h_{it}^{\text{com}}\}$'s (originating from the level-common shocks),

the role of Germany by the $\{h_{it}^{\text{idio}}\}$'s originating from the level-idiosyncratic shocks).

The strongly common, weakly common and weakly idiosyncratic components all qualify as "market-driven volatilities".

In the S&P100 case below, $q = q^{\text{com}} = q^{\text{idio}} = 1$ is identified. Then, the decomposition only has strongly common components and strongly idiosyncratic ones. Market volatility is univariate (one shock).

We illustrate the method by an application to the S&P100 series : $n = 90$ series [some stocks were not traded, and were removed from the analysis] of daily log-returns observed between January 2000 and September 2013 (T =3457).

Step 1. A factor model analysis of the levels Y_{it}

• a number $q = 1$ of dynamic factors is identified via the Hallin-Liška (*JASA* 2007) method

• the one-sided method of Forni-Hallin-Lippi-Zaffaroni (*Journal of Econometrics* 2015) yields (a reconstruction of) the level-common components X_{it}, their shocks e_{it}, and the level-idiosyncratic Z_{it}

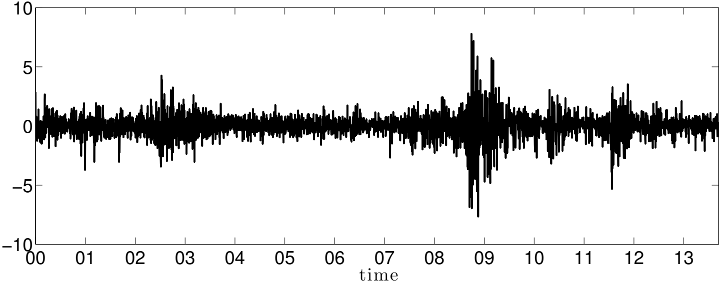
• univariate AR models (orders selected via AIC or BIC) are fitted to the Z_{it}'s, yielding residuals v_{it}

• the volatility proxies $\{h_{it}^{\mathbf{com}} := \log e_{it}^2\}$ are computed from the level-common shocks e_{it}

• the volatility proxies $\{h_{it}^{\mathbf{idio}} := \log v_{it}^2\}$ are computed from the level-idiosyncratic shocks v_{it}

Step 1.

Estimated market shocks $\widehat{u}_t$ on returns, period 2000–2013.



Note

- the dot–com bubble, the Enron (late 2001) and Worldcom (mid–2002) scandals
- the 2003 Iraq war
- the Great 2008–2009 Financial crisis starting with Lehman Brothers bankruptcy (September 2008);
- the 2010–2012 euro sovereign bond crisis.

The largest shocks over the period, by far, are those related with the 2008–2009 financial crisis.

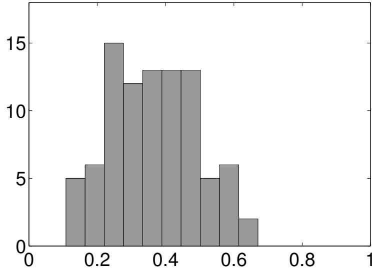
Step 1.

One can compute the ratios between the sum of the (empirical) variances of the estimated common components $\mathbf{X}_t^T$ and the sum of the (empirical) variances of the observed returns:

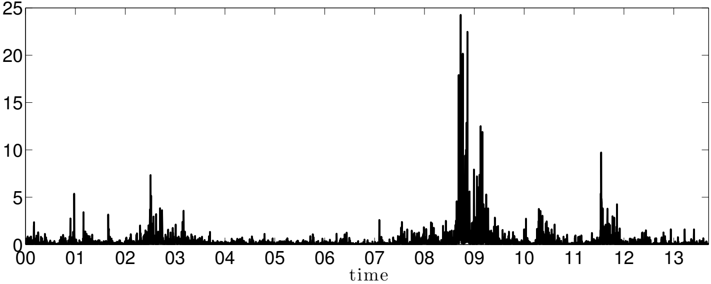
$$R^2_{Y.\text{market}} := \frac{\sum_{i=1}^n \sum_{t=1}^T (X_{it}^T)^2}{\sum_{i=1}^n \sum_{t=1}^T (Y_{it})^2}. \approx 0.36$$

and also

$$R^2_{Y_i.\text{market}} := \frac{\sum_{t=1}^T (X_{it}^T)^2}{\sum_{t=1}^T (Y_{it})^2}, \quad i = 1, \ldots, n \text{ and } R^2_{Y_t.\text{market}} := \frac{\sum_{i=1}^n (X_{it}^T)^2}{\sum_{i=1}^n (Y_{it})^2}, \quad t = 1, \ldots, T$$



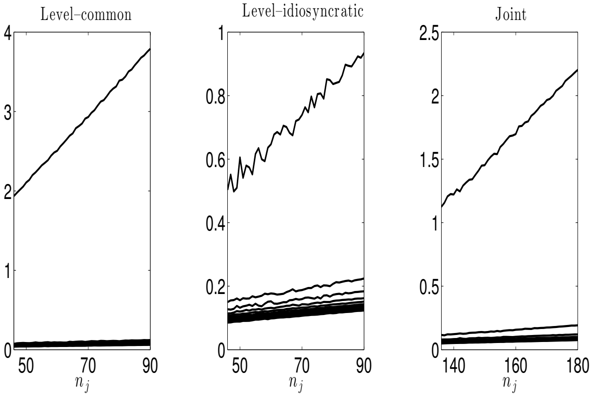
Histogram for the proportions $R^2_{Y_i.\text{market}}$ of variance explained by the market shocks to returns across the panel.



Time series of the proportions $R^2_{Y_t.\mathbf{market}}$ of variance explained by the market shocks to returns at time t.

Step 2. A 2-block factor model analysis of the volatility proxies $\{h_{it}^{\mathbf{com}}\}$ and $\{h_{it}^{\mathbf{idio}}\}$

Evidence of factor structure in the volatility proxy panels.



Ten largest dynamic eigenvalues, averaged over frequencies, computed for panels of increasing sizes: $45 \leq n_j \leq n = 90$ for the level–common and level–idiosyncratic volatility panels, and $135 \leq n_j \leq 2n = 180$ for the joint volatility panel.

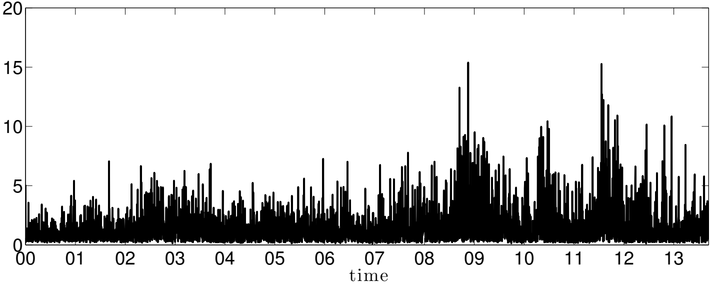
Step 2.

A 2-block factor model analysis of the volatility proxies $\{h_{it}^{\text{com}}\}$ and $\{h_{it}^{\text{idio}}\}$

• the following numbers of dynamic factors are identified via the Hallin-Liška (JASA 2007) method: $q^{\text{com}} = 1$, $q^{\text{idio}} = 1$, $q = 1$.

• This implies that a **unique volatility-strongly-common shock** is driving both the level-common h_{it}^{com}'s and the level-idiosyncratic h_{it}^{idio}'s: no weakly common nor weakly idiosyncratic components here, which greatly simplifies the analysis (a standard FHLZ approach to the $2n$-dimensional panel is sufficient)

• That common shock thus qualifies as *the* **market volatility shock**, impacting both the level-common and level-idiosyncratic components of the S&P100 panel, with different strengths, though



Estimated market shock on volatilities, period 2000–2013.

Note

- 01 the dotcom bubble
- 03 Iraq war
- 09 is the Great Financial Crisis (which started in 2008)
- 11-12 is the Eurocrisis

The overall contribution of market shocks to the variances of the volatility proxies $\{h_{it}^{\mathbf{com}}\}$ and $\{h_{it}^{\mathbf{idio}}\}$ can be evaluated by means of the ratios

$$R^2_{\text{com.market}} := \frac{\sum_{t=1}^{T}\sum_{i=1}^{n}(\phi_{\mathbf{com};it})^2}{\sum_{t=1}^{T}\sum_{i=1}^{n}(h_{it}^{\mathbf{com}})^2} \approx 0.60$$

and

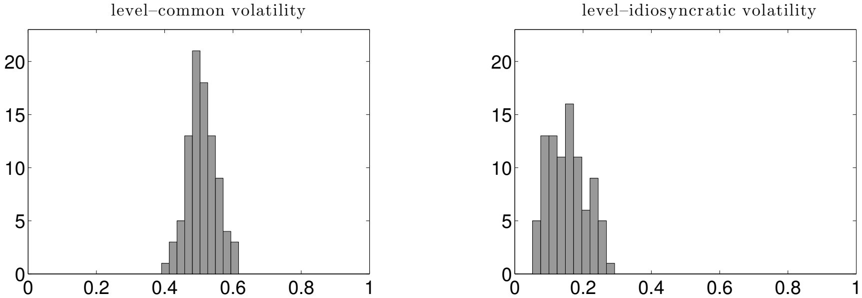
$$R^2_{\text{idio.market}} := \frac{\sum_{t=1}^{T}\sum_{i=1}^{n}(\phi_{\mathbf{idio};it})^2}{\sum_{t=1}^{T}\sum_{i=1}^{n}(h_{it}^{\mathbf{idio}})^2} \approx 0.17$$

For each individual stock i, a measure of the same impact is

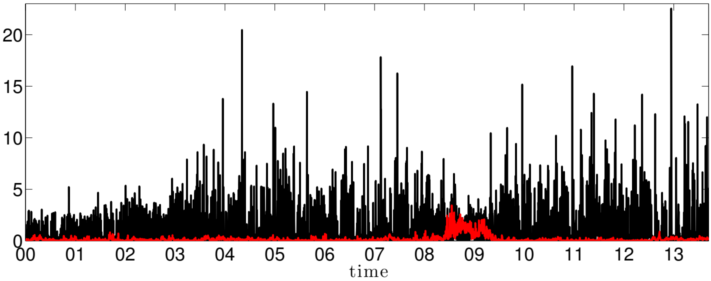
$$R^2_{h_i^{\mathbf{com}}.\text{market}} := \frac{\sum_{t=1}^{T}(\phi_{\mathbf{com};it})^2}{\sum_{t=1}^{T}(h_{it}^{\mathbf{com}})^2} \quad \text{and} \quad R^2_{h_i^{\mathbf{idio}}.\text{market}} := \frac{\sum_{t=1}^{T}(\phi_{\mathbf{idio};it})^2}{\sum_{t=1}^{T}(h_{it}^{\mathbf{idio}})^2}, \quad i = 1, \ldots, n;$$

while their evolution through time is captured by

$$R^2_{h_t^{\mathbf{com}}.\text{market}} := \frac{\sum_{i=1}^{n}(\phi_{\mathbf{com};it})^2}{\sum_{i=1}^{n}(h_{it}^{\mathbf{com}})^2} \quad \text{and} \quad R^2_{h_t^{\mathbf{idio}}.\text{market}} := \frac{\sum_{i=1}^{n}(\phi_{\mathbf{idio};it})^2}{\sum_{i=1}^{n}(h_{it}^{\mathbf{idio}})^2}, \quad t = 1, \ldots, T.$$

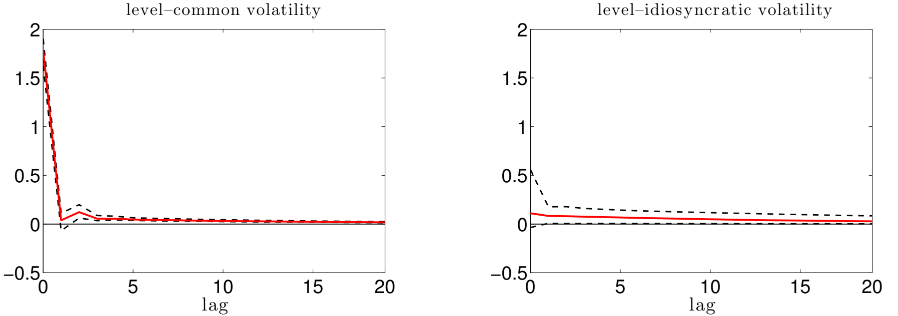


Histograms for the proportions of variances explained by the market volatility shocks across the panel: $R^2_{h_i^{\mathbf{com}}.\mathbf{market}}$ (left) and $R^2_{h_i^{\mathbf{idio}}.\mathbf{market}}$ (right).

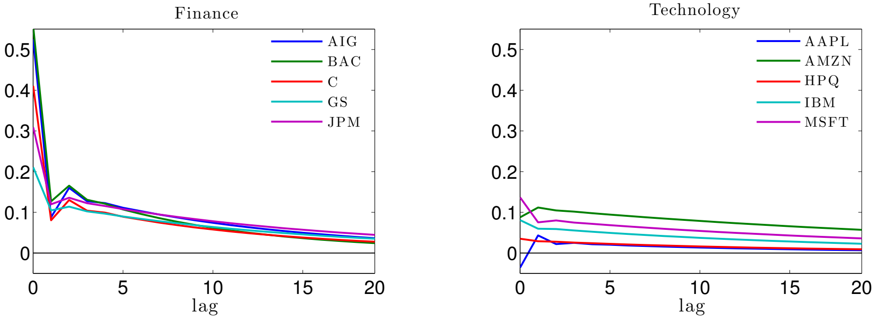


Time series of the estimated proportions $R^2_{h_t^{\mathbf{com}}.\mathbf{market}}$ (black) and $R^2_{h_t^{\mathbf{idio}}.\mathbf{market}}$ (red) of variances explained by the market volatility shocks.

The transfer or impulse–response functions describing the dynamic loading, by the volatility proxies, of the market volatility shocks. For each stock i, those functions take the form of scalar filters (one for $h_{it}^{\mathbf{com}}$, another one for $h_{it}^{\mathbf{idio}}$), plotted sequences of coefficients associated with the various lags.



Median, maximum, and minimum of the distribution of impulse–response functions of volatilities to a one–standard-deviation market volatility shock, that is, the sequence of loading coefficients divided by the standard error of the shocks, for level–common (left) and level–idiosyncratic (right) volatilities, respectively.

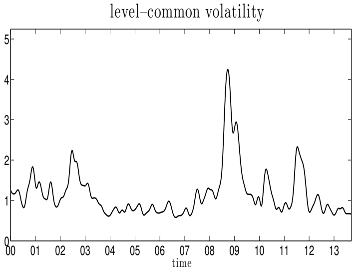


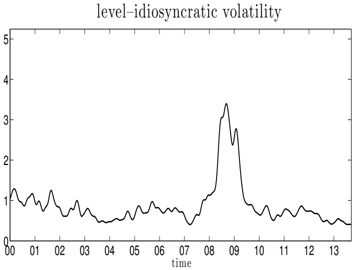
Impulse–response functions of volatilities to a one–standard-deviation market volatility shock, that is, the sequence of loading coefficients divided by the standard error of the shocks, for level–idiosyncratic volatilities of selected stocks from the Financial (left) and Technology (right) sectors, respectively.

Finally, to conclude, we turn to the analysis, for a few selected stocks, of the market-driven volatilities, which we define (hats omitted) as

$$\chi_{\mathbf{e^2};\mathbf{it}} := \exp(\phi_{\mathbf{com};it} + \bar{h}_i^{\mathbf{com}}), \quad \chi_{\mathbf{v^2};\mathbf{it}} := \exp(\phi_{\mathbf{idio};it} + \bar{h}_i^{\mathbf{idio}}), \ \ i = 1, \ldots, n, \quad t = 1, \ldots, T$$

where $\bar{h}_i^{\mathbf{com}}$ and $\bar{h}_i^{\mathbf{idio}}$ stand for empirical means.

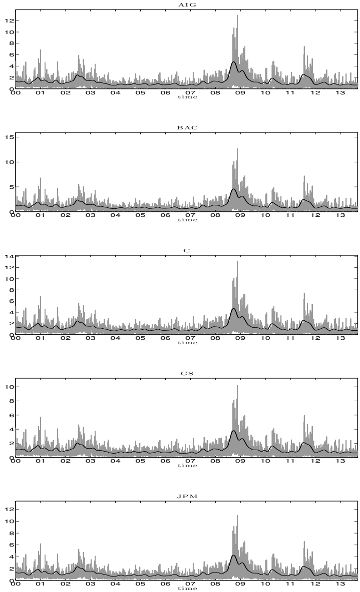




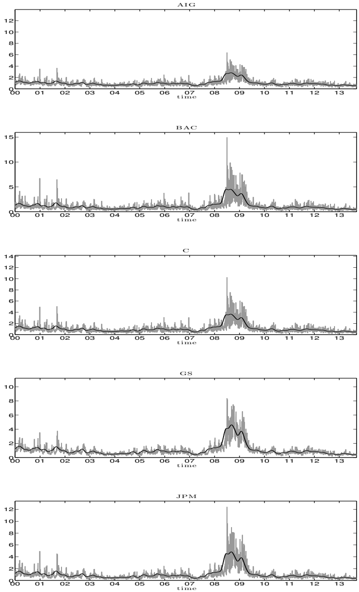
Kernel-smoothed cross–sectional averages of market volatilities. The bandwidth used corresponds to 3 weeks of trading (15 days).

Market volatilities – Financial sector.

level–common volatility

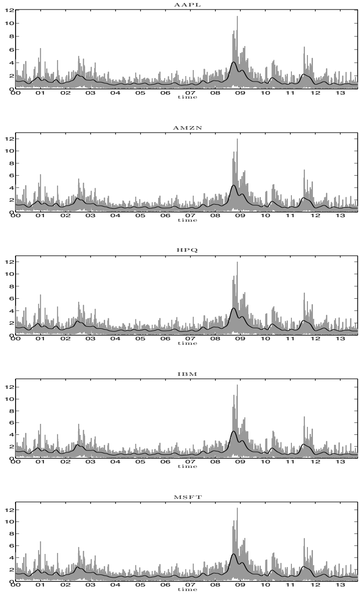


level–idiosyncratic volatility

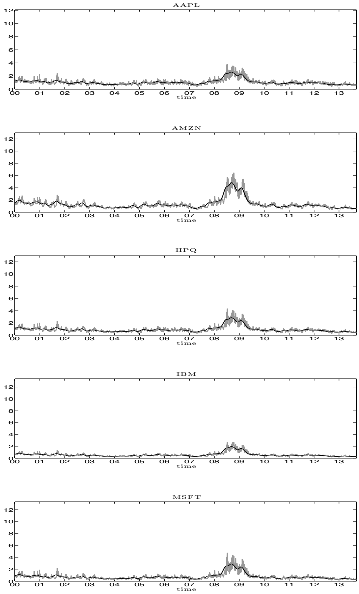


Market volatilities – Technology sector.

level–common volatility



level–idiosyncratic volatility



Conclusions and perspectives

• Dynamic factor methods can be applied to volatilities in high-dimensional time series (in large panels of stocks)

• contrary to most existing methods for the analysis of volatility, they are fully nonparametric and model-free: curse of dimensionality turns into a blessing!

• the decompositions between "level-common" and "level-idiosyncratic" on one hand, between "volatility-common" and "volatility-idiosyncratic" in general do not coincide: common volatility shocks quite significantly do affect level-idiosyncratic components as well as the level-common one;

• dynamic portfolio optimization should take into account the market impact on the volatilities of the level-idiosyncratic components a well as their impact on the level-common ones; in general, the risk associated with level-idiosyncratic components cannot be fully diversified away, while the risk associated with level-common partially can

This approach opens the door to volatility prediction and portfolio optimization without curse of dimensionality nor oversimplified modeling in large panels (high-dimensional time series) of stock returns.